Factorisation Algebraic Expressions

To factorise means to find the factors that made up a product, say 4 and 5 are the factors of 20 as they produce 20 when multiplied. So as 20 and 1 are other factors of 20, as well as 10 and 2.

Similarly, in the previous section, if $4(2x + 1) = 8x + 4$, we multiplied 4 and $(2x + 1)$ to produce the algebraic expression $8x + 4$. Hence, 4 and $(2x + 1)$ are called factors of $(8x + 4)$.

To factorise the expression $(14m – 21)$ means to find two expressions that will give such expression. If we think about it, a number must have been multiplied to an expression that resulted to $14m – 21$. The HCF of 14 and 21 is 7, hence, is this number. So, $7(2m – 3)$ is the factorised form of $14m – 21$.

The process of writing an algebraic expression as a product of two or more other algebraic expressions is called **factorisation**.

EXAMPLE 1:

Factorise the following:

(a) $9x + 24$ (*b*) $2m – 4n + 10p$ (*c*) $bx + by – bz$

Solution:

(a) $9x + 24$

$\mathbf{3} \times 3x + \mathbf{3} \times 8$

$3(3x + 8)$

(*b*) $2m – 4n + 10p$

$\mathbf{2} \times m – \mathbf{2} \times 2n + \mathbf{2} \times 5p$

$2(m – 2n + 5p)$

(*c*) $bx + by – bz$

$\boldsymbol{b} \times x + \boldsymbol{b} \times y – \boldsymbol{b} \times z$

$b(x + y – z)$

(d) $6ab + 18ac$ (e) $– 4x^2 + 36x$ (f) $25x^2y – 5xy$

Solution:

(d) $6ab + 18ac$

$\mathbf{6a} \times b + \mathbf{6a} \times 3c$

$6a(b + 3c)$

(e) $– 4x^2 + 36x$

$–\mathbf{4x} \times x + –\mathbf{4x} \times –9$

$– 4x (x – 9)$

(f) $25x^2y – 5xy$

$\mathbf{5xy} \times 5x – 5\mathbf{xy} \times 1$

$5xy (5x – 1)$

Some algebraic expressions contain factors in the form of a group of terms.
Say, factorise the expression $x(x + 1) + y(x + 1)$.
As can be observed, the group $(x + 1)$ must have been multiplied to another expression resulting the given expression. By "factoring out the common group", it turns out that the second expression is $x + y$.

Hence, $x\mathbf{(x + 1)} + y\mathbf{(x + 1)} = \mathbf{(x + 1)}\ (x + y)$

EXAMPLE 2:

Factorise:

(a) $k(5m + 2) + 3(5m + 2)$ (b) $4a(x - y) - 3b(x - y)$

Solution:

(a) $k(5m + 2) + 3(5m + 2)$

$\mathbf{(5m + 2)} \times k + \mathbf{(5m + 2)} \times 3$

$\mathbf{(5m + 2)\ (k + 3)}$

(b) $4a(x - y) - 3b(x - y)$

$\mathbf{(x - y)} \times 4a - \mathbf{(x - y)} \times 3b$

$\mathbf{(x - y)\ (4a - 3b)}$

PRACTICE:

1. Factorise the following fully:

(a) $4a + 16$ (b) $8k - 64$

(c) $-10p + 45q$ (d) $27 - 63w$

(e) $20dx + 60dy - 30dz$ (f) $6a - 30am + 12an$

(g) $3xyz - 15wxyz$ (h) $-4m + 8n - 12p$

(i) $-60r^2st^3 - 30rs^2t^2 - 40rs^3t$ (j) $2ax + 3ay - 5yz$

2. Factorise the following fully:

(a) $4(a + 1) + x(a + 1)$ (b) $5m(k - 2) - 3(k - 2)$

(c) $6x(a + b - c) - y(a + b - c)$ (d) $-2w(1 - b) + 5(1 - b)$

(e) $20h(x + 3y) - (x + 3y)$ (f) $(4 + z) - x(4 + z)$

(g) $3xy(2 - z) - 15x(2 - z)$ (h) $5x(a + 4b) + 10y(a + 4b)$

(i) $(x + y)(a + 4b) + (2x + y)\ (a + 4b)$ (j) $(6x - 1)(a + 4b - 3c) - 7y(a + 4b - 3c)$